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## Statistics and Probability Letters

journal homepage: [www.elsevier.com/locate/stapro](http://www.elsevier.com/locate/stapro)

# High order approximation for the coverage probability by a confident set centered at the positive-part James–Stein estimator

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## ARTICLE INFO

## Article history:

Received 7 September 2008

Received in revised form 9 May 2009

Accepted 11 May 2009

Available online 22 May 2009

## MSC:

primary 62E20

secondary 62F10

## ABSTRACT

In this paper we continue our investigation connected with the new approach developed in Ahmed et al. [Ahmed, S.E., Saleh, A.K.Md.E., Volodin, A., Volodin, I., 2006. Asymptotic expansion of the coverage probability of James–Stein estimators. *Theory Probab. Appl.* 51 (4) 1–14] for asymptotic expansion construction of coverage probabilities, for confidence sets centered at James–Stein and positive-part James–Stein estimators. The coverage probabilities for these confidence sets depend on the noncentrality parameter  $\tau^2$ , the same as the risks of these estimators. In this paper we consider only the confidence set centered at the positive-part James–Stein estimator. As is shown in the above-mentioned reference, the new approach provides a method to obtain for the given confidence set, an asymptotic expansion of the coverage probability as one formula for both cases  $\tau \rightarrow 0$  and  $\tau \rightarrow \infty$ . We obtain the third terms of the asymptotic expansion for both mentioned cases, that is, the coefficients at  $\tau^2$  and  $\tau^{-2}$ . Numerical illustrations show that the third term has only a small influence on the accuracy of the asymptotic estimation of coverage probability.

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## 1. Introduction

The problem of confidence estimation of the mean vector  $\theta = (\theta_1, \dots, \theta_p)$  for the  $p$ -dimensional normal distribution with independent components and equal variances  $\sigma^2 = 1$  is considered. Let  $\bar{X} = (\bar{X}_1, \dots, \bar{X}_p)$  be the vector of sample means calculated by samples of common size  $n$  from the marginal distributions. The confidence set

$$D_{\bar{X}} = \left\{ \theta : n \sum_{i=1}^p (\theta_i - \bar{X}_i)^2 \leq c^2 \right\}$$

has the given confidence coefficient  $1 - \alpha$ , if  $c^2$  is the quantile of chi-square distribution with  $p$  degrees of freedom given by the relation  $K_p(c^2) = 1 - \alpha$ , where  $K_p(\cdot)$  is the chi-square distribution function.

This confidence set possesses the minimax property, but there exist other minimax sets that obtain bigger coverage probability for all values of the noncentrality parameter  $\tau^2 = n\|\theta\|^2$  if  $p \geq 4$ . In this paper we consider one of these sets

$$D_{\delta^+} = \{ \theta : n\|\theta - \delta^+(\bar{X})\|^2 \leq c^2 \},$$

which is centered at the positive-part James and Stein (1961) estimator given by

$$\delta^+(\bar{X}) = \left( 1 - \frac{p-2}{n\|\bar{X}\|^2} \right) \bar{X} \mathbf{I}\{n\|\bar{X}\|^2 > p-2\}$$

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